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DEPTH OF PENETRATION OF SOLID PARTICLES INJECTED INTO A GAS FLOW

L. Kh. Garmize, V. K. Popov,
S. P. Fisenko, and S. I. Shabunya

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The depth of penetration of a spherical particle into a uniform transverse flow was determined experimentally and theoretically for both a continuous medium and a free molecule flow.

In solving a number of practical problems of two-phase hydrodynamics, it is often necessary to determine the depth of penetration of particles introduced into the flow.

Let us examine this problem for a continuous medium. The following classical expression for depth of penetration S is known for small Reynolds numbers Re $\ll$ 1:

$$S=\frac{\rho_p d^2 V_0}{18\mu}. \tag{1}$$

However, S is independent of the velocity of the transverse flow and, as was shown in [1], when Re > 1 substantial corrections of (1) are necessary to determine the penetration depth.

Let us choose a coordinate system such that the x axis will be perpendicular to the flow and the y axis will be parallel to the flow. Then the equations of motion of the particles will have the following form:

$$\frac{dx}{dt}=V_p, \tag{2}$$

$$\frac{dV_p}{dt}=-\frac{3C_D\,\mathrm{Re}\,\mu}{4\rho_p d^2}V_p, \tag{3}$$

$$\frac{dU_p}{dt}=\frac{3C_D\,\mathrm{Re}\,\mu}{4\rho_p d^2}(U_q-U_p), \tag{4}$$

with the initial conditions $t = 0$, $V_p = V_0$, $x = 0$, and $U_p = 0$. The quantity C_D is a function of the Reynolds number

$$\mathrm{Re}=\frac{\rho_q d\,[(U_q-U_p)^2+V_p^2]^{1/2}}{\mu}.$$

We will evaluate the braking distance on the basis of (2) and (3). The braking time τ and distance S are equal to the following, respectively, in order of magnitude

$$\tau\sim\frac{V_p}{\left.\frac{dV}{dt}\right|_{t=0}}=\frac{\rho_p d^2}{C_D(\mathrm{Re}_0)\,\mathrm{Re}_0\mu},$$

$$S\sim V_0\tau=\frac{\rho_p d^2 V_0}{C_D(\mathrm{Re}_0)\,\mathrm{Re}_0\mu}. \tag{5}$$

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Considering that $\frac{\rho_p}{\rho_q} \gg 1$, the exact expression for S may be written in the form

$$S = \frac{\rho_p d^2 V_0}{\mu} f(\mathrm{Re}_0). \tag{6}$$

Here, the form of the function $f(\mathrm{Re}_0)$ should be determined from either a numerical or a full-scale experiment.

The experimental unit (Fig. 1) consisted of a nozzle 1, a water supply system 2, and a device for introducing the particles into the flow 3. To measure S, we determined the trajectory of the particles in the gas flow by photographing them with the aid of a stroboscope. The frequency of the stroboscope was varied in relation to the velocity of the gas flow. We took for the braking distance S the projection, on the normal to the flow axis, of the section of the trajectory from the point of impact with the flow A to the lowest point of the trajectory B — where $V_p = 0$. The scale of the image was established by photographing a standard which was placed in the plane of impact of the particles. We ensured a uniform transverse flow by using the initial section of the planar turbulent jet, with the nozzle width having been many times greater than the particle diameter.

System of equations (2)-(4) was solved numerically. We used Klyachko's empirical formula $C_D = \frac{24}{\mathrm{Re}}\left(1 + \frac{1}{6}\mathrm{Re}^{2/3}\right)$ [2] to assign the explicit form of the function $C_D(\mathrm{Re})$, but assumed that $C_D = 0.4 = \mathrm{const}$ at $\mathrm{Re} \geqslant 10^3$ [3]. Figure 2a shows the function $f(\mathrm{Re}_0)$ obtained as a result of the experiment and numerical calculation. It can be seen that the braking distance is very slightly dependent on the Reynolds number Re at $\mathrm{Re}_0 \geqslant 2000$ and that the experimental results agree well with the numerical solution of system (2)-(4).

The penetration of a spherical particle into a transverse free molecule flow, when $\mathrm{Kn} = \lambda/d \gg 1$ [4], is also of interest. Here, it is natural to ignore the perturbing effect of the sphere on the state of the gas flow. We will examine first the penetration of the sphere into a quiescent gas. We will choose a coordinate system such that the x axis is directed along the direction of motion of the sphere. Then the equations of motion will have the form

$$\frac{dx}{dt} = V_p, \quad \frac{dV_p}{dt} = -\frac{6}{\pi\rho_p d^3} F_c, \tag{7}$$

with the initial conditions $t = 0$, $x = 0$, $V_p = V_0$. The quantity F_c is the drag, equal to the following [5]:

$$F_c = \frac{\rho_q k I \pi d^2}{4m} \left\{ \frac{e^{-\varepsilon^2}}{\sqrt{\pi}\,\varepsilon}(1 + 2\varepsilon^2) + \left(2\varepsilon^2 + 2 - \frac{1}{2\varepsilon}\right)\Phi(\varepsilon) \right\}.$$

It is assumed that the reflection of molecules from the sphere is ideally elastic in character. In this case, the depth of penetration of the particles is equal to the distance in which the velocity of the particles V_x falls from the initial value to the thermal velocity, equal to $\sqrt{\frac{6kT}{\pi d^3 \rho_p}}$.

At $\varepsilon_0 < 1$, the following analytical expression for depth of penetration can be obtained from the equations of motion

$$S = \frac{\rho_p V_0 d}{\rho_q} \sqrt{\frac{\pi m}{32kT}} \left(1 - \frac{mV_0^2}{30kT}\right). \tag{8}$$

The solution of the equations of motion of particle (7) are shown in Fig. 2b for $0 \leqslant \varepsilon_0 \leqslant 7$. It should be noted that if the reflection of the gas molecules from the sphere is diffuse in nature, then the depth of penetration is doubled. In the case of vaporized particles, the results obtained can serve as an estimate of the penetration depth — since S is directly proportional to d.

Let us examine the case where the gas is moving at a velocity U. We will change over to a coordinate system which is stationary with respect to the gas. Then the spherical particle will have a velocity which in absolute value is equal to $V_0 = \sqrt{U_q^2 + V_0^2}$. Here, $V_0/\tilde{V}_0 = \cos\varphi$. To determine the depth of penetration S_1, we can use the results already

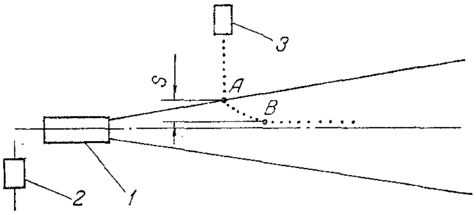

Fig. 1. Set-up of experiment.

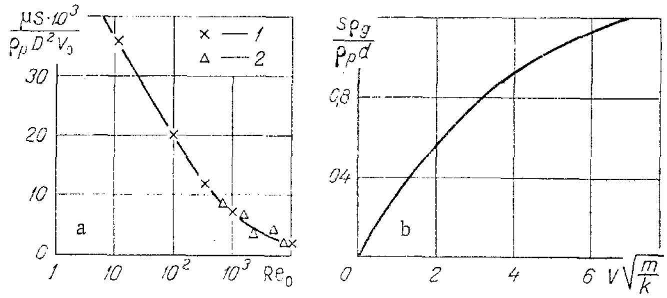

Fig. 2. Dependence of dimensionless depth of penetration of particles: a) on Reynolds number (1 — numerical calculation; 2 — experiment); b) on ratio of particle velocity to thermal velocity of molecules.

obtained for a quiescent gas but substitute $\tilde{V}_0$ for V_0. Then the penetration depth along the x axis will be equal to

$$S = S_1 \cos \varphi.$$

To determine the order of magnitude of the depth of penetration of solid particles into a gas flow, we obtained numerical estimates using the data in [4]. Thus, with $\rho_q = 5 \cdot 10^{-3}$ kg/cm³, T = 200°K, ρ_p = 500 kg/m³, U_q = 2500 m/sec, and d = 1 μm, the depth of penetration of the particles into the flow is $1.2 \cdot 10^{-3}$, $2.9 \cdot 10^{-3}$, and $5.9 \cdot 10^{-3}$ m for particle velocities V_0 = 200, 500, and 1000 m/sec, respectively.

NOTATION

ρ_p, particle density, kg/m³; d, particle diameter, m; μ, viscosity of gas, kg/m·sec; t, time, sec; V_p, particle velocity across the direction of flow of the gas, m/sec; U_p, particle velocity in the direction of flow of the gas, m/sec; ρ_q, density of gas; k, Boltzmann's constant; T, absolute temperature, °K; m, mass of gas molecule; α = 1, accommodation coefficient; λ, mean free path of gas molecules. $\varepsilon = V_p \Big/ \sqrt{\frac{2kT}{m}}$; $\Phi(\varepsilon) = \frac{2}{\sqrt{\pi}} \int_0^{\varepsilon} e^{-x^2} dx.$

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